



GSJ: Volume 6, Issue 8, August 2018, Online: ISSN 2320-9186
www.globalscientificjournal.com

Transition probability of particle's Quantum State

Author, Samuel Mulugeta Bantikum

University of Gondar, Department of Physics.

Abstract

This study mainly focused on calculating the transition probability of a particles in a given quantum state based on the idea of quantum jump, since Quantum particles can change their quantum state very quickly. By Using perturbation theory method, the approximation probability transition of particle's quantum state is calculated interms of time and Time dependent Schrodinger equation is solved by a systematic approach. Finally, the probability of transition of particles were expressed interms of time.

Keywords

Schrodinger equation, Quantum state, Quantum jump, Hamiltonian, Time dependent perturbation theory(TDPT), Two-level systems.

Introduction

Quantum particles can change their state very quickly, this called quantum jump. An atom or particles can absorb a photon thereby changing in to a state of high energy. A particle change its quantum state because of external factor like small perturbation. Quantum state refers to the state of a quantum system and quantum state allow the system to be in a few states simultaneously otherwise particles are Jumps from one state to another state. If asystem exposed to external effect,a particle will forced to change thier quantum state. The transiton of thier quantum state is depend up on time, then the probability of particle quantum state should be expressed interms of time and the dynamics parts of quantum particles are given by Time dependent schrodinger equation as below.

0.1 Time-dependent Perturbation theory

If we consider a quantum dynamics with potentials do not depend on time, the Schrodinger equation is Given by

$$H\psi = i\hbar \frac{\partial \psi}{\partial t} \quad (1)$$

Where, $\psi(r, t) = \psi(r)e^{-\frac{iEt}{\hbar}}$ and $\psi(r)$ satisfies $H\psi(r) = E\psi(r)$

If we want to consider transitions (quantum jumps) between different energy levels, we need to introduce quantum dynamics. We will focus on the

particular important problem (the emission and absorption of radiation by an atom). To treat effectively the quantum jump, let us consider a system that contains two different subsystem, which is called two level system and stated below.

0.1.1 Two-level systems(Quantum system with two subsystems

In this quantum state a quantum particles has a possibility of transition (quantum jump) from one subsystem to another subsystem with a certain probability of transition. So to calculate the probability of transition of particle's quantum state in terms of time.

We can start with a quantum system that has only two orthonormal states ψ_a and ψ_b that are eigenstates of the unperturbed Hamiltonian.

$$\begin{aligned} H^0\psi_a &= E_a\psi_a, & H^0\psi_b &= E_b\psi_b, \\ \langle\psi_a|\psi_b\rangle &= \delta_{ab} \end{aligned}$$

Of course, any state can be expressed as the linear combination of these two states.

$$\psi(t=0) = c_a\psi_a + c_b\psi_b \quad (2)$$

If there is no time-dependent perturbation, the time evolution of such state is described by

$$\psi(t) = c_a\psi_a e^{-\frac{iE_a t}{\hbar}} + c_b\psi_b e^{-\frac{iE_b t}{\hbar}} \quad (3)$$

$$\text{Where, } |c_a|^2 + |c_b|^2 = 1$$

Where $|c_a|^2$ is the probability of a particle to be at a quantum state ψ_a and $|c_b|^2$ is the probability of a particle to be at a quantum state ψ_b

Now, we turn on the time-dependent perturbation, described by the Hamiltonian H' .

The resulting wave function can still be represented as the superposition of our two unperturbed functions, since they make a complete set, but the coefficients c now depend on time:

$$\psi(t) = c_a(t)\psi_a e^{-\frac{iE_a t}{\hbar}} + c_b(t)\psi_b e^{-\frac{iE_b t}{\hbar}} \quad (4)$$

What happens during the transition between these two energy levels?

If the system was originally in state a, then $c_a(t=0) = 1$ and $c_b(t) = 0$ since the $|c_a|^2$ and $|c_b|^2$ are the corresponding probabilities that measurement of energy will give results E_a and E_b . As a result of the transition at time t , $c_a(t) = 0$ and $c_b(t) = 1$.

Our mission: to determine the coefficients $c_a(t)$ and $c_b(t)$.

First, we derive the equations for $c_a(t)$ and $c_b(t)$. To accomplish that, we substitute the wave function

$$\psi(t) = c_a\psi_a e^{-\frac{iE_a t}{\hbar}} + c_b\psi_b e^{-\frac{iE_b t}{\hbar}}$$

into the Schrödinger equation $H\psi = i\hbar \frac{\partial \psi}{\partial t}$

with $H = H^0 + H'(t)$, then we get:

$$[H^0 + H'(t)] \left[c_a\psi_a e^{-\frac{iE_a t}{\hbar}} + c_b\psi_b e^{-\frac{iE_b t}{\hbar}} \right] = i\hbar \frac{\partial}{\partial t} \left[c_a\psi_a e^{-\frac{iE_a t}{\hbar}} + c_b\psi_b e^{-\frac{iE_b t}{\hbar}} \right] \quad (5)$$

$$\begin{aligned} & c_a H^0 \psi_a e^{-\frac{iE_a t}{\hbar}} + c_b H^0 \psi_b e^{-\frac{iE_b t}{\hbar}} + c_a H' \psi_a e^{-\frac{iE_a t}{\hbar}} + c_b H' \psi_b e^{-\frac{iE_b t}{\hbar}} \\ &= i\hbar \frac{\partial}{\partial t} \left[c_a\psi_a e^{-\frac{iE_a t}{\hbar}} + c_b\psi_b e^{-\frac{iE_b t}{\hbar}} \right] \end{aligned}$$

Note that ψ_a and ψ_b do not depend on time, as they are eigenstates of the unperturbed Hamiltonian H^0 and similar terms are cancel out and we get:

$$c_a H' \psi_a e^{-\frac{iE_a t}{\hbar}} + c_b H' \psi_b e^{-\frac{iE_b t}{\hbar}} = i\hbar \dot{c}_a \psi_a e^{-\frac{iE_a t}{\hbar}} + i\hbar \dot{c}_b \psi_b e^{-\frac{iE_b t}{\hbar}} \quad (6)$$

Next step: separate this equation into two, one containing \dot{c}_a and another one containing \dot{c}_b . To get the first equation for \dot{c}_a , multiply this equation

on the left by ψ_a^* and integrate, i.e. take inner product with ψ_a . Use inner product designations. Then, use orthogonality of the ψ_a and ψ_b wave functions.

solve the above equation for \dot{c}_a

$$c_a \langle \psi_a | H' \psi_a \rangle e^{-\frac{iE_a t}{\hbar}} + c_a \langle \psi_a | H' \psi_b \rangle e^{-\frac{iE_b t}{\hbar}} = i\hbar \dot{c}_a \langle \psi_a | \psi_a \rangle e^{-\frac{iE_a t}{\hbar}} + i\hbar \dot{c}_b \langle \psi_a | \psi_b \rangle e^{-\frac{iE_b t}{\hbar}}$$

$$i\hbar \dot{c}_a e^{-\frac{iE_a t}{\hbar}} = c_a H'_{aa} e^{-\frac{iE_a t}{\hbar}} + c_b H'_{ab} e^{-\frac{iE_b t}{\hbar}}$$

If $\omega_0 = \frac{E_b - E_a}{\hbar}$, we will get the value of \dot{c}_a :

$$\dot{c}_a = \frac{\partial c_a(t)}{\partial t} = -\frac{i}{\hbar} \left[c_a H'_{aa} + c_b H'_{ab} e^{-\omega_0 t} \right] \quad (7)$$

Similarly, To get equation for \dot{c}_b , take inner product with ψ_b .

The resulting equation is:

$$\dot{c}_b = \frac{\partial c_b(t)}{\partial t} = -\frac{i}{\hbar} \left[c_b H'_{bb} + c_a H'_{ba} e^{\omega_0 t} \right] \quad (8)$$

For most cases, the diagonal matrix elements of the perturbation Hamiltonian H' are zero:

$$H'_{bb} = H'_{aa} = 0 \quad (9)$$

The resulting equations become:

$$\dot{c}_a = -\frac{i}{\hbar} H'_{ab} e^{-\omega_0 t} c_b \quad (10)$$

$$\dot{c}_b = -\frac{i}{\hbar} H'_{ba} e^{\omega_0 t} c_a \quad (11)$$

These equations are exact for the cases where ($H'_{bb} = H'_{aa} = 0$) is true as we made no approximations so far. However, there are very few problems which can be solved exactly. If the perturbation is small, we can use time-dependent perturbation theory.

We take the perturbation $H'(t)$ to be small. We start with our system in the ground state, ψ_a . Therefore, the values of the coefficients are the following:

$$\boxed{c_a(t=0) = 0} \text{ and } \boxed{C_b(1) = 1}$$

Zeroth-order

If there is no perturbation, then our system just stays in this state forever. Therefore, our zeroth-order is (superscript ⁽⁰⁾ designating the zeroth order):

$$\boxed{c_a^{(0)} = 1} \text{ and } \boxed{c_b^{(0)} = 0} \quad (12)$$

First order

To get the first-order approximation for the coefficients $c_a(t)$ and $c_b(t)$, we take our zeroth-order values and substitute them into right side of the equations that we have just derived.

First order for $c_a(t)$

$$\begin{aligned} \dot{c}_a &= -\frac{i}{\hbar} H'_{ab} e^{-\omega_0 t} c_b \\ &= -\frac{i}{\hbar} H'_{ab} e^{-\omega_0 t} c_b^{(0)} = 0 \\ \frac{dc_a^{(1)}(t)}{dt} &= 0 \Rightarrow c_a^{(1)}(t) = 1 \end{aligned}$$

Therefore, first order for $c_a(t)$ is given by:

$$\boxed{c_a^{(1)}(t) = 1} \quad (13)$$

First order for $c_b(t)$:

$$\begin{aligned} \dot{c}_b &= -\frac{i}{\hbar} H'_{ba} e^{\omega_0 t} c_a \\ &= -\frac{i}{\hbar} H'_{ba} e^{-\omega_0 t} c_a^{(0)} = -\frac{i}{\hbar} H'_{ba} e^{-\omega_0 t} \\ \frac{dc_b^{(1)}(t)}{dt} &= -\frac{i}{\hbar} H'_{ba} e^{-\omega_0 t} \end{aligned}$$

Therefore, first order for $c_b(t)$ is given by:

$$\boxed{c_b^{(1)}(t) = -\frac{i}{\hbar} \int_0^t H'_{ba}(t') e^{\omega_0 t'} dt'} \quad (14)$$

Second-order

To get second-order approximation, repeat the same thing: plug in your first-order result

$$c_a^{(1)}(t) = 1$$

$$c_b^{(1)}(t) = -\frac{i}{\hbar} \int_0^t H'_{ba}(t') e^{\omega_0 t'} dt'$$

into the right size of the general form of the equations for coefficients c :

$$\dot{c}_a = -\frac{i}{\hbar} H'_{ab} e^{-\omega_0 t} c_b$$

$$\dot{c}_b = -\frac{i}{\hbar} H'_{ba} e^{\omega_0 t} c_a$$

Finally after substitution, we get:

The Second order for $c_b(t)$ is given by:

$$\boxed{c_b^{(2)}(t) = c_b^{(1)}(t)} \quad (\text{since } c_a^{(1)}(t) = 1) \quad (15)$$

Therefore, The Second order for $c_b(t)$ is given by:

$$\boxed{c_a^{(2)}(t) = 1 - \frac{1}{\hbar^2} \int_0^t H'_{ab}(t') e^{-\omega_0 t'} \left[\int_0^{t'} H'_{ba}(t'') e^{\omega_0 t''} dt'' \right] dt'} \quad (16)$$

0.2 Results and Conclusions

Quantum particles can change their state very quickly. Particles in a quantum system that has subsystems are jumps from one state to other and the sum of all probability at each subsystem will give us one. The transition Probability of a particle in a given quantum state to be in state ψ_a is given by first order, second order, third order of $C_a(t)$ and The transition Probability of a particle in a given quantum state to be in state ψ_b is given by first order, second order, third order of $C_b(t)$, If we do more order, we will get a better result.

0.3 Reference

1. John S. Townsend, A Modern Approach to Quantum Mechanics, 2nd University Science Books, (2000)

VOLUME 6, ISSUE 8, August 2018

2. W. Greiner, Quantum Mechanics (An Introduction), 4 th ed., Springer (2008).
3. David Griffith, Introduction to Quantum Mechanics: Benjamin Cummings, (2004).
4. J. J. Sakurai, Modern Quantum Mechanics Revised edition, (1993).
5. R. Shankar, Principles of Quantum Mechanics, 2 nd ed., (2008)
6. J. Singh, Quantum Mechanics: Fundamentals and Applications to Technology 1 st ed., (1996).
7. David A.B. Miller, Quantum Mechanics for Scientists and Engineers, (2008).
8. Byron, F. W. and Fuller, R. W. (1969) Mathematics of Classical and Quantum Physics. Addison-Wesley, Reading, Massachusetts.
9. B. H. Brandsen and C. J. Joachain, Quantum Mechanics, 2 nd ed., Benjamin Cummings, (2000)
10. John Taylor, Modern Physics for Scientists and Engineers, Pearson Education, (2003).